

Inference at \*  
of proof for Lemma exists\_functionality\_wrt\_iff:

$\vdash \forall S, T: \text{Type}, P, Q: (S \rightarrow \mathbb{P}).$   
 $(S = T) \Rightarrow (\forall x: S. P(x) \iff Q(x)) \Rightarrow ((\exists x: S. P(x)) \iff (\exists y: T. Q(y)))$   
by ((GenUnivCD)  
CollapseTHENA ((Auto\_aux (first\_nat 1:n) ((first\_nat 1:n), (first\_nat  
3:n)) (first\_tok :t) inil\_term))).

1:

1.  $S : \text{Type}$
  2.  $T : \text{Type}$
  3.  $P : S \rightarrow \mathbb{P}$
  4.  $Q : S \rightarrow \mathbb{P}$
  5.  $S = T$
  6.  $\forall x: S. P(x) \iff Q(x)$
  7.  $\exists x: S. P(x)$
- $\vdash \exists y: T. Q(y)$

2:

1.  $S : \text{Type}$
  2.  $T : \text{Type}$
  3.  $P : S \rightarrow \mathbb{P}$
  4.  $Q : S \rightarrow \mathbb{P}$
  5.  $S = T$
  6.  $\forall x: S. P(x) \iff Q(x)$
  7.  $\exists y: T. Q(y)$
- $\vdash \exists x: S. P(x)$